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STUDY OF AN ECONOMIC ORDER QUANTITY MODEL UNDER LINEAR DEMAND, CONSTANT DETERIORATION AND SHORTAGES ARE PERMITTED

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ABSTRACT

Economic Order Quantity (EOQ) model is developed to minimize the total Inventory cost. In this paper, an Economic Order Quantity (EOQ) model is developed for deteriorating items with linear demand pattern and constant deterioration rate. We permit shortages. The aim of this model is to check the convexity of the graph between total inventory cost and time and to calculate the sensitivity analysis of the optimal solution.

KEYWORDS: *Deterioration, Constant Deterioration Rate, Economic Order Quantity (EOQ), Demand, Linear Demand*

Article History

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INTRODUCTION

Inventory is stocks of consuming goods. It is of two types

- Direct inventory – The items which play a direct role in the manufacturing and is the main part of complete goods is added indirect inventory.
- Indirect inventory – Materials which are necessary for the manufacturing of items but not a part of complete goods are included in the indirect inventory.

In many departmental stores, the manager has to maintain the stock which depends on many factors like time of order, demand, lead time, etc. So, they deal with problems like under-stocking and over-stocking. Inventory should be maintained for the smooth and efficient running of any business firms so that they can provide best services to customers immediately or at a short time span. In inventory, EOQ is the order quantity which maintains minimum total inventory cost. It involves costs like holding cost, shortage cost, deterioration cost and etc. This is known as the EOQ model because it gives the most economic size of the order to place. Witn in begun research on inventory and gave a theory about the deterioration of fashion items after a certain time of storage. Later, Ghare and Schrader started their study for the consumption of deteriorating items which was close to the negative exponential function of time. With the help of linear differential equation, they stated deterioration of inventory which included inventory level and demand rate as a function of time. Shah and Jaiswal considered the demand to be a function of time and presented an ordered level inventory model for deteriorating items. Their model was rectified by Aggarwal. These models assumed deterioration rate and demand pattern to be constant and replenishment was assumed to be infinite and shortages were denied. In 1963, continuously decaying inventory for a constant demand was considered by Ghare and Schrader for the first time. Then, the variable deterioration rate of two parameters of Weibull distribution was used to formulate a model which assumed constant demand rate and shortages were denied by R.B.Covert and G.S.Philip in 1973. Then Philip extended their model for variable deterioration rate. However, all these models were limited as they were only applicable to constant demand. In 2013, Trilokyanath Singh and Hadibandhu Pattnayak worked on an EOQ Model for deteriorating items which assumed linear demand, variable deterioration and partial backlogging. Further, in 2015, they worked on an ordering policy with time-proportional deterioration, linear demand and permissible decay in payment and again in 2016 with Pandit Jagatananda Mishra, they developed an EOQ Model for deteriorating items with linear demand, salvage value and partial backlogging. EOQ models are used in various managing firms like food managing firms, merchandise managing firms, and power plants and etc. The objective of this model is to check the convexity of the graph between total inventory cost and time and to calculate the sensitivity analysis of the optimal solution with the help of a numerical example.

Assumption

We took the following assumptions in this model.

- The demand rate is Linear and time-dependent , $D(t)= c+dt$,
- Where c is a constant fraction of demand and b is the fraction of demand which varies with time. c, d>0 and c is primary demand.
- Shortages are permitted with complete backlogging.
- The lead time is zero.
- Rate of Replenishment is infinite.
- Deterioration rate is constant i.e. θ

Symbols used

We have used the following symbols in this Model

C_h: Holding cost per unit time.

C_s: Shortage cost.

C_o: Ordering cost of inventory per order.

C_d: Deteriorating cost per unit.

t_1: time when shortages start.

T: length of each ordering cycle.

W: the maximum inventory level for each ordering cycle [0, T].

S: the maximum amount of demand backlogged for each ordering cycle i.e. [t_1, T].

Q: the economic order quantity for each ordering cycle.

I (t): the inventory level at time t.

TC: Total Inventory cost per unit time.

Illustration

The replenishment is made at time t=0 and at this interval the inventory level is maximum i.e. W. In this interval [0, t_1}, the demand and deterioration of items in inventory level decline linearly and then falls to zero at time t=t_1. Then interval [t_1,shortages occur and at time t=T reaches the maximum inventory level i.e. S

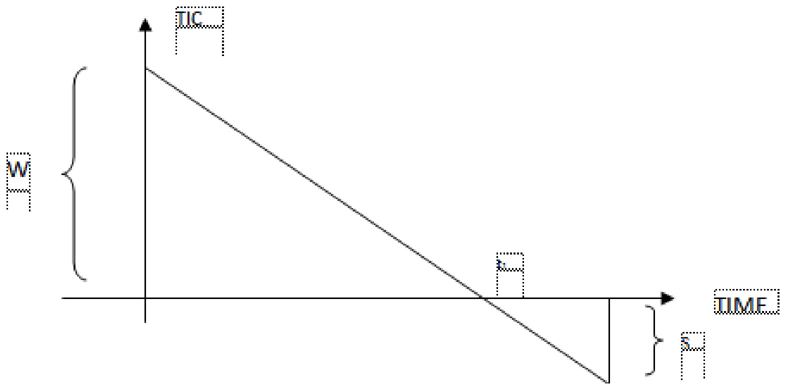

Figure 1

$$\frac{dI_1}{dx} + \theta\ I_1(t) = -D(t) \quad ;0 \ \leq \ t \ < t_1 \tag{1}$$

where D(t) = c + dt.

The solution of eq[n] (1) using the condition $I_1(t_1) = 0$ is

$$I_1(t)=\left[c\left\{(t_1 - t) + \frac{\theta(t_1^2-t^2)}{2}\right\} + d\left\{\frac{(t_1^2-t^2)}{2} + \frac{\theta(t_1^3-t^3)}{3}\right\}\right] - \theta\left[c\left\{(t_1 t - t^2) + \frac{\theta\ (t_1^2 t-t^3)}{2}\right\} + d\left\{\frac{(t_1^2 t-t^3)}{2} + \frac{\theta(t_1^3 t-t^4)}{3}\right\}\right] \tag{2}$$

(higher power is neglected of θ as $0< \theta \ll 1$).

Max Inventory level is calculated by putting the boundary conditions $I_1(0) = W$ in eq[n] (2). We have,

$$I_1(0)=W=\left[c\left(t_1 + \frac{\theta {t_1}^2}{2}\right) + d\left(\frac{{t_1}^2}{2} + \frac{\theta {t_1}^3}{3}\right)\right] \tag{3}$$

During the shortage interval [t_1, T], the demand at time t is completely backlogged.

Therefore the equation for the amount backlogged is

$$\frac{dI_2}{dx} = -(c+dt) \quad ; t_1 \ \leq \ t \leq \ T \tag{4}$$

with the boundary condition $I_2\ (t_1) = 0$

The solution of Equation (4) is

$$I_2\ (t) = \ c(t_1 - t) + \frac{d(t_1^2-t^2)}{2} \tag{5}$$

Max amount of backlogged demand is calculated by substituting t = T in Eq[n] (5). We have,

$$S = -I_2(T) = -c(t_1 - t) + \frac{d(t_1^2 - t^2)}{2} \quad (6)$$

And, the Economic Order Quantity is

$$Q = W + S = \left[c\left(T^2 + \frac{\theta t_1^2}{2}\right) + d\left(\frac{T^2}{2} + \frac{\theta t_1^3}{3}\right)\right] \quad (7)$$

The holding cost is

$$HC = C_h \int_0^{t_1} I_1(t)dt$$

$$= C_h\left[c\frac{t_1^2}{2} + \left(\frac{c\theta + 2d}{6}\right)t_1^3 + \left(\frac{d\theta - c\theta^2}{8}\right)t_1^4 - \frac{d\theta^2}{10}t_1^5\right] \quad (8)$$

higher power is neglected of θ as $0 < \theta \ll 1$).

The shortage cost is

$$SC = -C_s \int_{t_1}^{T} I_2(t)dt$$

$$= -C_s\left[c\left(t_1 T - \frac{T^2}{2}\right) + \frac{d}{2}\left(t_1^2 T - \frac{T^3}{3}\right) - c\frac{t_1^2}{2} - d\frac{t_1^3}{3}\right] \quad (9)$$

The opportunity cost is

$$OC = C_o \quad (10)$$

The deteriorating cost is

$$DC = C_d\left[W - \int_0^{t_1} D(t)dt\right]$$

$$= C_d\left[c\frac{\theta\, t_1^2}{2} + d\frac{\theta\, t_1^3}{3}\right] \quad (11)$$

Therefore, the total inventory cost per unit time per cycle= (deteriorating cost+ shortage cost + holding cost+ ordering cost+ opportunity cost)/length of the ordering cycle ,i.e.

$$TC = \frac{1}{T}\left[C_h\left[\frac{ct_1^2}{2} + \left(\frac{(c\theta+2d)t_1^3}{6}\right) + \left(\frac{(d\theta - c\theta^2)t_1^4}{8}\right) - \left(\frac{d\theta^2 t_1^5}{10}\right)\right] - C_s\left[c\left(t_1 T - \frac{T^2}{2}\right) + \frac{d}{2}\left(t_1^2 T - \frac{T^3}{3}\right) - \frac{ct_1^2}{2} - \frac{dt_1^3}{3}\right] + C_o +\right.$$

Cdθt122+dθt133

(12)

We aspire to evaluate the optimal (best) values of t_1 and T in order to reduce the total inventory cost TC to a minimum.

Example

Now, we will see an example based on the above theory

Example: Values of Parameters in the inventory system be as given below:

c=30, d=15, C_h=50, C_s=1.5, C_o=50, C_d=.5 and $\theta = .005$

On solving we have, optimal shortage period t_1 = 0.0368 unit time and length of ordering cycle T = 1.26362 unit time. And the economic order quantity is

Q = 59.87774763 units and the minimum total inventory cost per unit time

TC = 73.15493756. We get a graph according to the above values:

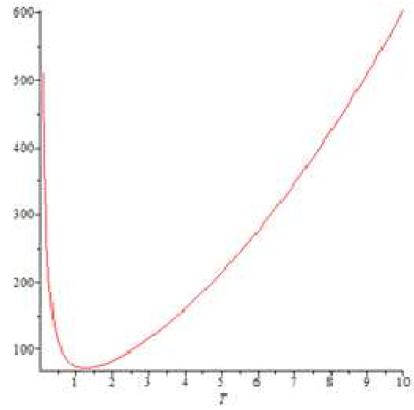

Figure 2

Table 1

Parameter	TC %Change in Parameter	t	T	Q	TC	%Change in TC
C	-20%	0.03922	1.34674	57.13199	67.45764	-7.788
	-10%	0.03796	1.30348	58.6176	70.35164	-3.832
	10%	0.03573	1.22679	60.95309	75.87453	3.717
	20%	0.03473	1.19266	61.87596	78.51667	7.329
d	-20%	0.03783	1.29899	60.74516	71.92532	-1.68
	-10%	0.03729	1.28074	60.2808	72.54861	-0.828
	10%	0.03633	1.24751	59.5277	73.74554	0.766
	20%	0.03589	1.23229	59.22379	74.32149	1.594
C_h	-20%	0.04577	1.26647	60.14835	72.95245	-0.276
	-10%	0.04079	1.26489	59.99803	73.06471	-0.123
	10%	0.03352	1.26258	59.77931	73.22904	0.101
	20%	0.03077	1.26172	59.69727	73.29099	0.185
C_s	-20%	0.03256	1.38919	72.36949	66.19795	-9.509
	-10%	0.03474	1.32149	65.48729	69.78257	-4609
	10%	0.03876	1.21339	55.21253	76.34606	4.362
	20%	0.04063	1.16925	51.26821	79.37999	8.509
C_o	-20%	0.03339	1.14641	49.28462	64.85879	-11.34
	-10%	0.03515	1.20698	54.63043	69.10762	-5.532
	10%	0.03835	1.31692	65.03593	77.02987	5.296
	20%	0.03982	1.36736	70.11272	80.75508	10.389
C_d	-20%	0.0368	1.26362	59.87775	73.15494	0
	-10%	0.0368	1.26362	59.87775	73.15494	0
	10%	0.0368	1.26362	59.87775	73.15494	0
	20%	0.0368	1.26362	59.87775	73.15494	0
θ	-20%	0.0368	1.26362	59.87775	73.15494	0
	-10%	0.0368	1.26362	59.87775	73.15494	0
	10%	0.0368	1.26362	59.87775	73.15494	0
	20%	0.0368	1.26362	59.87775	73.15494	0

Sensitivity Analysis

Now effects on the optimal cost are observed when the values of the parameters c, d, C_h, C_s, C_o, C_d and θ are changed. Each parameter is changed by $+10\%, +20\%, -10\%$ and -20% to observe sensitivity analysis and we take one parameter in one observation and rest of the parameters was kept same.

We have done our analysis by observing above example and the results are shown in table1 .We observed the following points.

- If we increase the value of parameter c then t_1 & T decreases while TC & Q increases.
- If we increase the value of parameter d then t_1, Q & T decreases while TC increases.
- If we increase the value of parameter C_h then t_1, Q & T decreases while TC increases.
- If we increase the value of parameter C_s then t_1 & TC increases while T & Q decreases.
- If we increase the value of parameter C_o then t_1, T, TC & Q also increases.
- If we increase the value of parameter C_d then t_1, T, TC & Q remains constant.
- If we increase the value of parameter θ then t_1, T, TC & Q remains constant.

CONCLUSIONS

Here, we conclude that the above EOQ Model is suitable for items having constant deterioration rate. We can use this model for things like clothes, books and furniture which has constant deterioration rate with time. Here, we have taken linear demand pattern. Further, by simultaneously optimizing the shortage period and length of cycle, we have used an example (numerical) which has minimum total inventory cost. At last, sensitivity analysis has been studied for all the parameters on the effect of optimal solution.

Further, investigation can be done on this research in a no. of direction like we can extend this model for different demand (exponential, binomial, passion, quadric and etc.).

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